



Brief Communication

Prediction of holdup, axial pressure gradient and wall shear stress in wavy stratified and stratified/atomization gas/liquid flow

N.A. Vlachos, S.V. Paras, A.J. Karabelas*

Department of Chemical Engineering and Chemical Process Engineering Research Institute, Aristotle University of Thessaloniki, Univ. Box 455, GR 540 06, Thessaloniki, Greece

Received 16 July 1997; received in revised form 21 July 1998

1. Introduction

The stratified flow regime is frequently encountered in long distance transfer pipelines (e.g. steam and water, natural gas and oil flows) and in power generation, petrochemical and process plants. Therefore, reliable estimates of hydrodynamic properties associated with this type of flow are considered essential for the safe design and efficient operation of two-phase systems. A method is proposed in this Note (using recent research results obtained by the authors) for predicting liquid holdup, frictional pressure loss and liquid-to-wall shear stress, averaged over the fraction of the pipe circumference covered by the continuous liquid phase.

Several empirical correlations and phenomenological models have been proposed for the prediction of stratified gas/liquid flow parameters over the past two decades. These models usually reflect, and are limited by, the current state of knowledge of the subject. Among the main drawbacks in modeling efforts are the assumption that the liquid-to-wall shear stress is nearly uniform around the pipe circumference (e.g. Taitel and Dukler, 1976; Fisher and Pearce, 1979), and the neglect of the influence of disturbance waves in calculating the gas/liquid interfacial shear stress (e.g. Agrawal et al., 1973).

Another issue of significance is the consideration regarding the shape of the surface between the two phases. In most of the models applied to stratified flow, the profile of the gas/liquid interface is assumed to be flat, as shown in Fig. 1(a) (e.g. Taitel and Dukler, 1976, Andritsos and Hanratty, 1987, Kowalski, 1987 etc.). Hart et al. (1989) assume a uniform shape of liquid film [Fig. 1(b)] in their apparent rough surface (ARS) model. However, Paras et al. (1994) and Vlachos et al. (1997) have confirmed by visual observations, that the area of the gas/liquid

* Corresponding author. Tel.: 30 31 996201/996 244; Fax: 30 31 996 209/474 121; E-mail: karabaj@alexandros.cperi.forth.gr

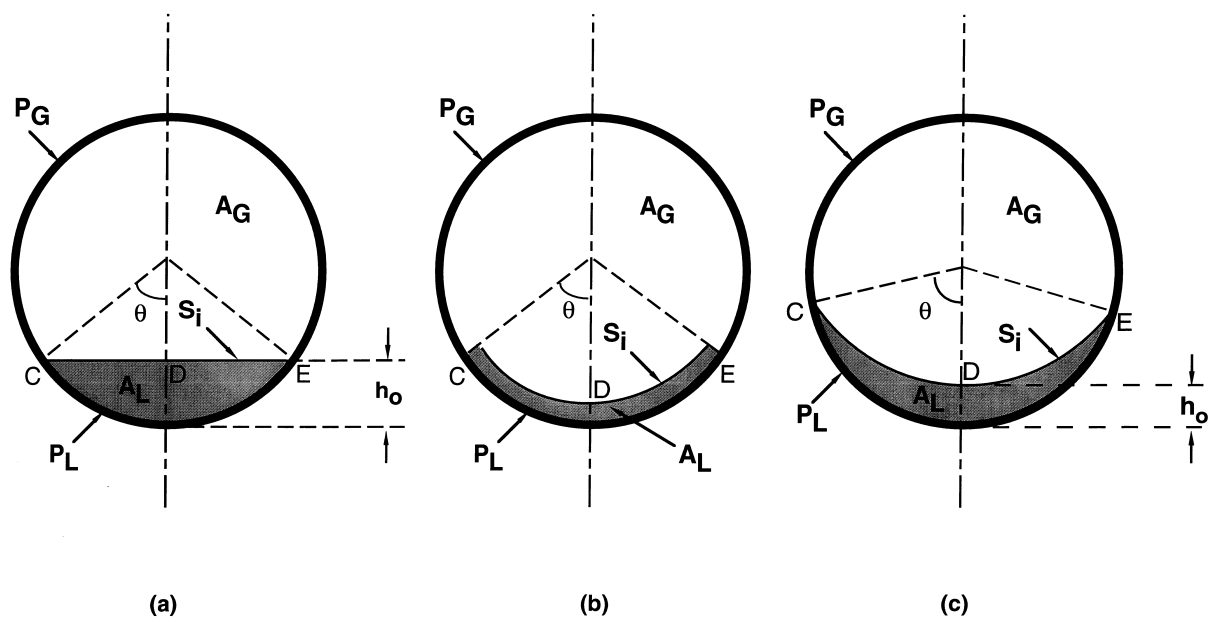


Fig. 1. Schematic representation of horizontal stratified two-phase pipe flow: (a) flat gas/liquid interface; (b) uniform liquid film shape (Hart et al., 1989); and (c) concave gas/liquid interface.

interface tends to increase with increasing gas velocity, and to deviate significantly from the flat (time-averaged) shape, as shown in Fig. 1(c). A similar consideration regarding the shape of the gas/liquid interface was taken into account by Grolman and Fortuin (1995), in their modified apparent rough surface (MARS) model.

In this R&D Note, an attempt is made to remove the aforementioned limitations and/or simplifications and to propose a computational method, taking advantage of the authors recent research results (Paras et al., 1994 and Vlachos et al., 1997). The latter include detailed liquid-to-wall shear stress measurements, liquid layer thickness and gas/liquid interface wave data, as well as pressure drop measurements and visual observations. Although the expressions employed to represent the above data are still empirical, they are presented here in the hope that (with further improvements) they will provide a more sound basis for predictions than other literature approaches relying on untested assumptions. In the next section the relevant literature is outlined. Following that, the computational procedure is presented and comparisons between measured and predicted values are made to assess the model performance.

2. Background literature

According to stratified flow phenomenological models, basic hydrodynamic parameters are estimated by solving one-dimensional liquid and gas momentum balance equations (e.g. Taitel

and Dukler, 1976; Chermisinoff and Davis, 1979; Andritsos and Hanratty, 1987 etc.):

$$-A_L(dp/dx) - \tau_{wL}P_L + \tau_i S_i = 0, \tag{1}$$

$$-A_G(dp/dx) - \tau_{wG}P_G - \tau_i S_i = 0 \tag{2}$$

where the geometrical parameters A_L , A_G , P_L , P_G , S_i are defined in Fig. 1. Implicit in the formulation of (1) and (2) is the key assumption of an equal axial pressure gradient (dp/dx) in both phases, considering steady state stratified flow in horizontal pipes with no hydraulic gradient present. In order to solve simultaneously these equations it is necessary to independently determine the liquid and gas phase geometrical parameters, the gas-to-wall and liquid-to-wall shear stresses (τ_{wG} and τ_{wL} respectively), and the interfacial shear stress, τ_i .

In most studies, the stress exerted by the gas flow on the pipe wall, τ_{wG} , is given in terms of the gas velocity and density and of a wall friction factor, f_G . The latter is determined from correlations applicable to single phase pipe flow (e.g. Blasius equation), introducing the concept of a hydraulic diameter.

In the often quoted model of Taitel and Dukler (1976), the same approach (using a Blasius type equation), was adopted for the determination of the liquid-to-wall shear stress, τ_{wL} , as well. However, Andritsos and Hanratty (1987) reported that τ_{wL} was better predicted via a characteristic stress τ_C (taken as the weighted average of τ_{wL} and τ_i). According to the authors, the characteristic stress can be calculated from a dimensionless liquid film height, h^+ , which is a known function of the liquid Reynolds number.

Using hot film probes, Kowalski (1987) made wall shear stress measurements on the liquid side as well as similar measurements at the gas/wall interface. He also measured Reynolds shear stresses in the gas phase. Based on his data obtained at relatively low gas velocities, Kowalski correlated the liquid-to-wall friction factor, f_L , with the liquid holdup, ϵ_L , and the liquid Reynolds number, Re_L , based on the superficial velocity and the pipe diameter.

As pointed out in the introduction, in currently used computational procedures, τ_{wL} is arbitrarily assumed to be uniformly distributed around the pipe circumference. However, Vlachos et al. (1997) by making detailed measurements of liquid-to-wall shear stress at various lateral positions, have shown that there is a significant shear stress circumferential variation. On the basis of these data, the following exponential expression was proposed to represent the circumferential variation of the time-averaged τ_{wL} :

$$\frac{\tau_{wL}(\Theta)}{\tau_{wG}} = 1 + \left(\frac{\tau_{wL0}}{\tau_{wG}} - 1 \right) \left\{ 1 - \exp\left(-m \frac{\theta - \Theta}{\Theta} \right) \right\} \tag{3}$$

where τ_{wL0} is the liquid-to-wall shear stress at the pipe bottom ($\Theta = 0^\circ$); θ is defined in Fig. 1; m is a dimensionless parameter. The stress τ_{wG} is considered to be constant, over the tube perimeter in contact with the gas phase (P_G), and equal to the liquid-to-wall shear stress value at the angle θ . The parameter m in equation (3) was determined by usual regression methods and found to be strongly influenced by both gas and liquid superficial velocities, U_G and U_L :

$$m = C_1 U_G^{-2} U_L^{-0.4}; \quad C_1 = 70(\text{m/s})^{2.4}. \tag{4}$$

It is pointed out that this correlation is based only on the available data obtained with low viscosity liquids and relatively small pipe diameters.

The shear stress at the pipe bottom, τ_{wL0} , and the real liquid velocity $U_{L,r}$ are used to define a friction factor f_{L0} :

$$\tau_{wL0} = f_{L0} \frac{\rho_L U_{L,r}^2}{2} \quad (5)$$

where ρ_L is the liquid density and $U_{L,r} = U_L/\varepsilon_L$.

Values of f_{L0} , obtained from (5), are fitted satisfactorily with a Blasius type equation:

$$f_{L0} = 0.2 Re_{LF}^{-0.25} \quad (6)$$

where

$$Re_{LF} = \frac{U_L h_0}{\varepsilon_L \nu_L};$$

h_0 is the time-averaged film thickness at the pipe bottom and ν_L the liquid kinematic viscosity.

The interfacial shear stress, τ_i , and the associated friction factor, f_i , are essential elements of stratified flow modeling. Taitel and Dukler (1976) assumed that the interfacial friction factor is equal to the gas-to-wall friction factor, i.e. $f_i = f_G$. This is hardly the case for wavy stratified flows and consequently this approach gives poor results. Among others, Spedding and Hand (1995) proposed a modification to the Taitel & Dukler model by assuming that $f_i/f_G = 4$ and 0.6, for turbulent–turbulent and turbulent–laminar gas-liquid flows, respectively.

Andritsos and Hanratty (1987) suggest that the ratio of friction factors, f_i/f_G , is almost unity if there are no roll waves at the gas/liquid interface. According to these authors, roll waves appear above a critical superficial gas velocity, $U_{G,t}$, estimated to be ~ 5 m/s for atmospheric pressure. For the case where $U_G > U_{G,t}$ they propose the following correlation:

$$f_i/f_G = 1 + 15 \left(\frac{h_0}{D} \right)^{0.5} \left(\frac{U_G}{U_{G,t}} - 1 \right). \quad (7)$$

Kowalski (1987) related his f_i data for the wavy stratified regime to the liquid holdup and to gas and liquid Reynolds numbers, Re_{GD} and Re_{LD} respectively, based on the real phase velocities and the pipe diameter. For the case of smooth stratified flow, the interfacial friction factor was correlated with the superficial gas Reynolds number alone.

Developing the ARS model for the horizontal gas/liquid pipe flow with small values of liquid holdup ($\varepsilon_L \leq 0.06$), Hart et al. (1989) use the following correlation for the interfacial friction factor (Eck, 1973):

$$f_i = 0.0625 / \left[\log_{10} \left(\frac{15}{Re_{GD}} + \frac{k}{3.715D} \right) \right]^2 \quad (8)$$

where k/D is the apparent relative roughness of the liquid film, taken as

$$k/D \cong 2.3 \left(\frac{\varepsilon_L}{4\theta'} \right). \quad (9)$$

The parameter θ' [equal to θ/π , Fig. 1(b)] is the wetted wall fraction and is related to the liquid holdup and to a modified Froude number of the liquid phase, as follows:

$$\theta' = 0.52\epsilon_L^{0.374} + 0.26Fr^{0.58} \quad (10)$$

The liquid Froude number is defined by

$$Fr = \frac{U_{L,r}^2 \rho_L}{gD \Delta\rho} \quad (11)$$

where $\Delta\rho = \rho_L - \rho_G$ and g is the acceleration due to gravity.

Based on their experimental data, Hart et al. correlated the ratio of liquid-to-wall friction factor to the interfacial friction factor, f_L/f_i , with the superficial liquid Reynolds number:

$$f_L/f_i = 108Re_L^{-0.726}. \quad (12)$$

Grolman and Fortuin (1995) proposed an improved MARS model utilizing liquid holdup and axial pressure gradient data obtained in separated gas/liquid flow in three different pipe diameters (i.e. 15, 26 and 51 mm i.d.), with angles of inclination ranging from $-3^\circ \leq \beta \leq +6^\circ$. They use an iterative procedure for predicting the interfacial friction factor; the required relative roughness, k/D , is obtained via a friction number Fn while making use of (8) for f_i . For the parameter θ' the authors suggest a correlation involving the liquid phase Weber and gas phase Froude numbers. It should be pointed out that for cases where the inclination angle $\beta = 0^\circ$ (i.e. horizontal pipe flow) (10) is valid for the MARS model as well.

Finally, Vlachos et al. (1997), employ averaged liquid-to-wall shear stress data, obtained from measured local values around the wetted portion of the pipe circumference, and complement them with data on liquid film thickness, wave properties and pressure drop measurements, to propose the following correlation for the interfacial friction factor:

$$f_i = 0.024\epsilon_L^{0.35} Re_L^{0.18} \quad (13)$$

For their calculations the gas/liquid interface is considered to be concave, which is verified by visual studies and film thickness measurements.

3. Computational procedure

The method proposed here applies to the case of steady, fully developed wavy stratified and stratified/atomization gas/liquid flow in horizontal pipes. An equal pressure gradient in the gas and liquid phases is assumed which is the convergence criterion for the method used. The computational procedure based on one-dimensional momentum balances for both phases ((1) and (2)) requires, as an input, the pipe diameter, fluid properties (i.e. density and viscosity), and gas and liquid superficial velocities (U_G , U_L).

Data obtained in the horizontal wavy stratified and stratified/atomization flow regimes by Hoogendoorn (1959), (air/gas-oil in a 140 mm i.d. pipe), by Andritsos (1986), (air/water flow in 25.2 and 95.3 mm i.d. pipes), by Paras et al. (1994), (air/water flow in a 50.8 mm i.d. pipe) and

by Vlachos et al. (1997), (air/ferri-ferrocyanide solution flow in a 24.0 mm i.d. pipe), were used to assess the performance of the model. With low viscosity liquids in relatively small diameter pipes, the gas/liquid interface deviates significantly from the commonly assumed flat profile. But, for the cases of stratified two-phase flow in large diameter pipes or with low gas flow rates, the assumption of the flat interface is shown to be realistic by film thickness measurements and visual studies (Andritsos, 1986; Vlachos, 1997). Thus, depending on the pipe diameter and on the gas flow rate a different approach is followed for the calculation of the phase geometrical parameters (Fig. 1), as outlined below.

In view of the above, a flat shape is assumed for the gas/liquid interface for the data corresponding to the 50.8 mm i.d. pipe with relatively low superficial gas velocities (i.e. $U_G < 15$ m/s) or to larger diameter pipes (i.e. 95.3 and 140 mm i.d.), where the degree of liquid climbing up the pipe side walls was observed to be insignificant. Using a bisection method, a value is sought of film thickness at the pipe bottom, h_0 (considering that under the conditions tested $0 < h_0 < D/2$), for which the difference between the pressure gradients in the gas and liquid phases equals zero (i.e. $(dP/dx)_G - (dP/dx)_L = 0$). The procedure for the calculation of the pressure gradients is described below:

1. Calculate all the gas and liquid phase geometrical variables [Fig. 1(a)] and the liquid holdup as simple functions of the film thickness at the pipe bottom, h_0 .
2. Compute the gas-to-wall shear stress using a friction factor f_G , as follows:

$$\tau_{wG} = f_G \frac{\rho_G U_{G,r}^2}{2} \quad (14)$$

where $f_G = 0.046 Re_G^{-0.2}$ and $U_{G,r} = U_G / (1 - \varepsilon_L)$ is the real gas velocity. In cases with relatively small liquid holdup values (such as those covered in this paper, e.g. $\varepsilon_L < 0.12$) one can assume that $U_{G,r} \approx U_G$.

3. Calculate the interfacial shear stress, τ_i , expressed in terms of an interfacial friction factor, f_i :

$$\tau_i = f_i \frac{\rho_G (U_{G,r} - U_i)^2}{2} \quad (15)$$

where U_i is the average interfacial velocity that can be approximated as $U_i = U_L / \varepsilon_L$. For the flow regimes examined here, the value of U_i is less than 10% of the real gas velocity, $U_{G,r}$. This estimate is supported by wave celerity measurements made by Andritsos (1986) and Paras et al. (1994). Consequently, considering also the complexity of the interfacial friction factor, (15) can be simplified by eliminating U_i . Moreover, as already mentioned, in cases with relatively small liquid holdup values $U_{G,r} \approx U_G$. For the determination of the interfacial friction factor use is made of (13).

4. From the momentum balance in the gas phase (2) calculate the pressure drop, $(dP/dx)_G$.
5. Calculate the liquid-to-wall shear stress, averaged over the fraction of the pipe circumference covered by the continuous liquid phase, by integrating Eq. (3) from $\Theta = 0^\circ$ to $\Theta = \theta$ and using (4)–(6) for the estimation of the τ_{wL0} and of the fitting parameter m .
6. From the momentum balance in the liquid phase (1) calculate the pressure drop, $(dP/dx)_L$.

For the data obtained in the 50.8 mm i.d. pipe with relatively high superficial gas velocities (i.e. $U_G > 15$ m/s) or in the 24.0 and 25.2 mm i.d. horizontal pipes, the shape of the gas/liquid interface is concave rather than flat [Fig. 1(c)]. Using a bisection method, a value is sought of liquid holdup, ϵ_L (considering that under the conditions tested $0 < \epsilon_L < 0.5$), for which the difference between the pressure gradients in the gas and liquid phases equals zero (i.e. $(dP/dx)_G - (dP/dx)_L = 0$). The procedure for the calculation of the pressure gradients is as follows:

1. Based on the liquid holdup, compute the real phase velocities and the parameter θ' (or the angle θ , $\theta' = \theta/\pi$), using (10) and (11) proposed by Hart et al. (1989). It should be emphasized that the predicted values of θ (from these equations), are in fairly good agreement with observed values by Paras et al. (1994) and Vlachos et al. (1997).
2. Estimate the film thickness at the pipe bottom using the following empirical correlation (Vlachos, 1997):

$$h_0 = C_2 D \frac{U_L^{0.35}}{U_G^{0.65}}; \quad C_2 = 1.5(\text{m/s})^{0.3}. \tag{16}$$

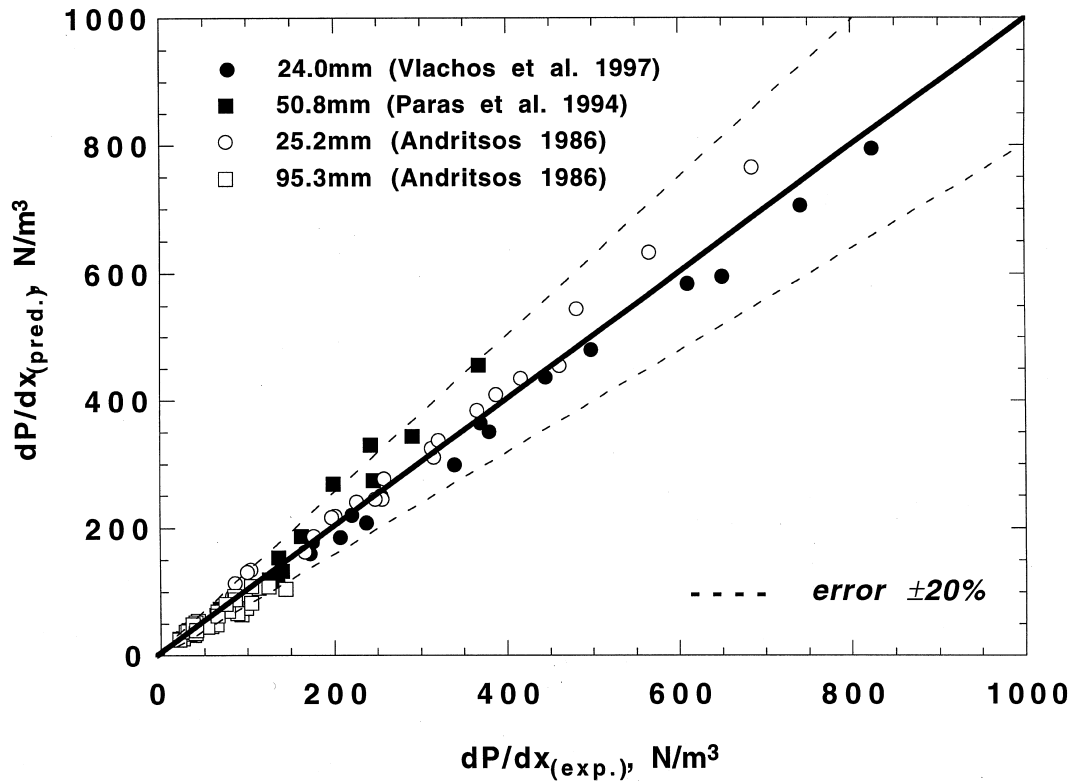


Fig. 2. Comparison between experimental and predicted values of axial pressure gradient.

This correlation reflects the trend of experimental data and provides first estimates for h_0 .

(3) Calculate the phase geometrical parameters based on the ε_L , θ , and h_0 values. The interfacial area S_i is approximated by taking the straight line segment CD instead of the arc CD (Fig. 1).

To proceed in the axial pressure gradient calculations for both phases, one can follow steps 2–6 of the previous case (i.e. flat gas/liquid interface).

3.1. Model validation

Experimental and predicted values of axial pressure gradient are compared in Fig. 2. The predictions are quite satisfactory with a maximum error less than 20%. Fig. 3 shows the model prediction for the liquid-to-wall shear stress, averaged over the fraction of the pipe circumference covered by the continuous liquid film. It turns out that there is a good agreement (max. error < 20%) between predicted and measured values of τ_{wL} , obtained in the 50.8 and 24.0 mm i.d. horizontal pipes. It should be pointed out that published data on mean

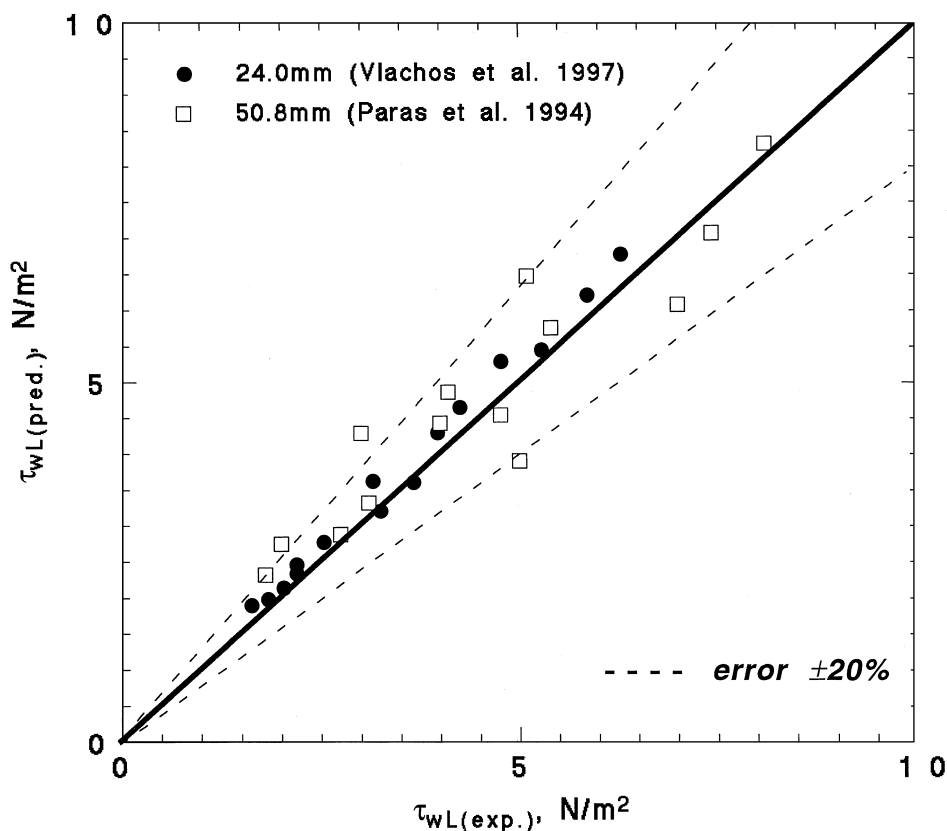


Fig. 3. Comparison between experimental and predicted values of liquid-to-wall shear stress, averaged over the fraction of the pipe circumference covered by the continuous liquid phase.

τ_{wL} for the stratified pipe flow (obtained from accurate measurements at various circumferential locations), are rather limited, although they are considered very helpful for improving our physical understanding and for modeling purposes.

Spedding and Hand (1995) evaluating the performance of stratified flow models for the prediction of holdup and pressure drop (using data obtained for air/water flow in 45.4 and 93.5 mm i.d. horizontal pipes), conclude that the best result is achieved with the ARS model of Hart et al. (1989). Lacking reliable liquid holdup measurements (corresponding to the rest of the data employed here), Fig. 4 presents a comparison between predicted holdup values from the ARS model and from the computational procedure recommended in this work. A similar comparison regarding axial pressure gradient values is showed in Fig. 5. The observed agreement is considered satisfactory with maximum errors less than 15% and 20% for the holdup and pressure drop, respectively. Especially for the case of small liquid holdup values, the agreement between the predictions of the two models is even better (Fig. 4) and is attributed to the fact that the ARS model was developed for ϵ_L values less than 0.06.

It is noted that the axial pressure gradient values in the air/gas–oil wavy flow reported by Hoogendoorn (1959), were obtained in a fairly large pipe diameter; i.e. 140 mm i.d. Andritsos

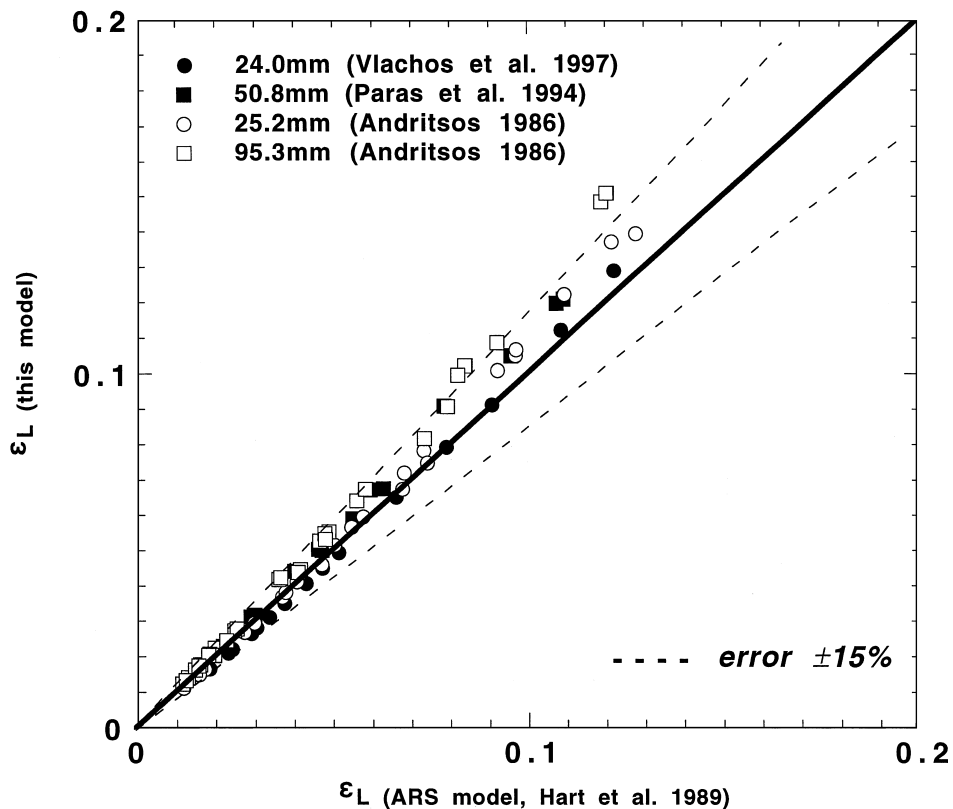


Fig. 4. Liquid holdup values predicted from this model in comparison with those calculated with the ARS model of Hart et al. (1989).

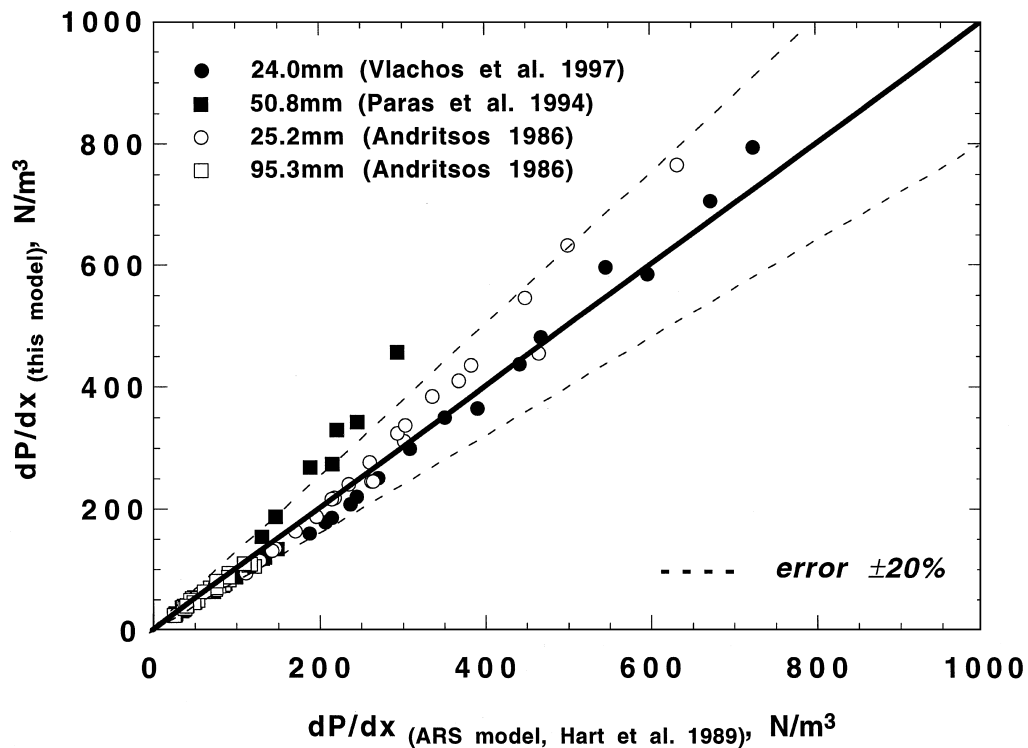


Fig. 5. Axial pressure gradient values predicted from this model in comparison with those calculated with the ARS model of Hart et al. (1989).

(1986) used these data for comparison with the values predicted by a model he proposed and found very good agreement. He also tested the predictions of the Taitel and Dukler (1976) and of Chermisinoff and Davis (1979) models. According to this author, the former model was found to underestimate the measured values whereas the latter to overpredict them. Fig. 6 shows the predictions of the method presented in this paper and of the ARS model proposed by Hart et al. (1989), for the same data obtained by Hoogendoorn. In this figure, the predictions of the Andritsos model are also included. It is clear that the agreement between the experimental pressure drop values and the predicted ones by the model proposed here, is satisfactory, whereas the ARS model predicts slightly higher pressure losses.

4. Concluding remarks

In this paper, a computational approach based on momentum balances for both phases is recommended for the prediction of liquid holdup, axial pressure gradient and average liquid-to-wall shear stress, for the wavy stratified and stratified/atomization gas/liquid flow in horizontal pipes. The performance of the model appears to be satisfactory and fair predictions

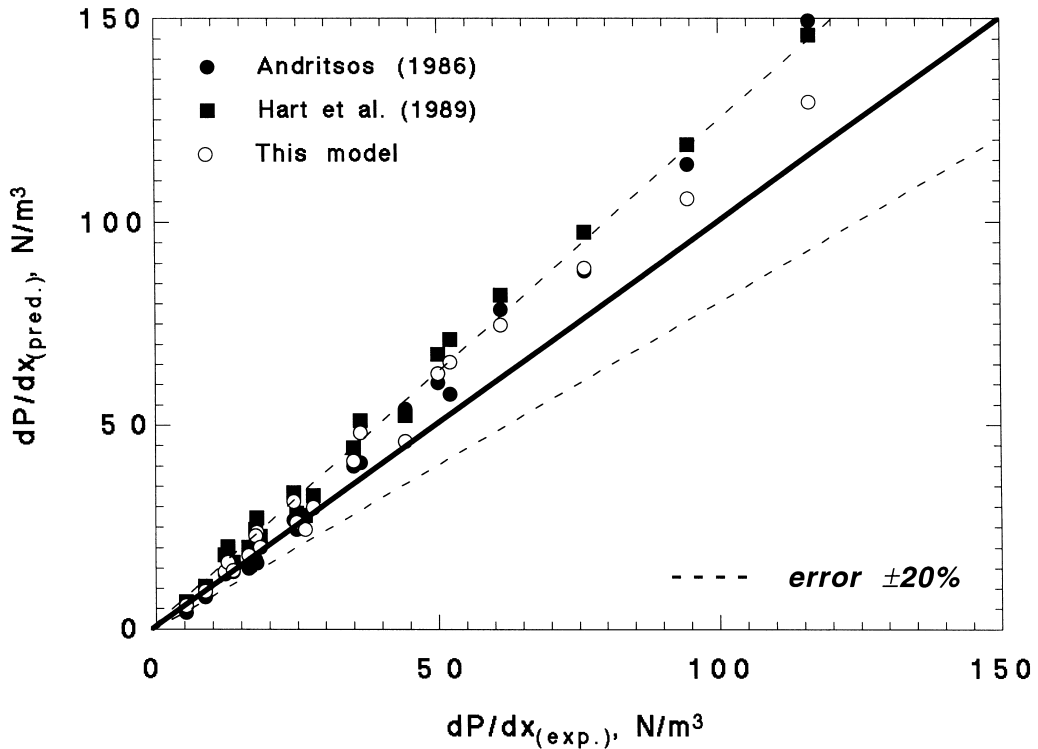


Fig. 6. Experimental axial pressure gradient data by Hoogendoorn (1959) plotted against the predictions of various models.

of the above two-phase parameters are obtained for all the data sets employed to assess its accuracy. The generally successful predictions are attributed mainly to:

1. The use of an improved method [(3)–(6)] for the determination of the average stress exerted by the liquid flow on the pipe wall, which is different from the often used Blasius type equation and based on τ_{wL} data obtained from detailed measurements of circumferential wall stress distribution.
2. The consideration regarding the shape of the gas/liquid interface and its possible distortion from the commonly assumed flat profile.
3. The use of (13) for the estimation of the interfacial friction factor, developed by utilizing detailed measurements of two-phase flow characteristics and visual observations.

The proposed method, and associated relations, although empirical are considered a useful computational tool. However, additional testing and improvements should be pursued, especially for two-phase flow systems with large pipe diameters or operating at high pressures.

Acknowledgements

Partial financial support by the European Commission under contract JOU2-CT92-0108 is gratefully acknowledged.

References

- Agrawal, S.S., Gregory, G.A., Govier, G.W., 1973. An analysis of horizontal stratified two phase flow in pipes. *Can J. Chem. Eng.* 51, 280–286.
- Andritsos, N., 1986. Effect of pipe diameter and liquid viscosity on horizontal stratified flow. Ph.D. Thesis, University of Illinois, Urbana.
- Andritsos, N., Hanratty, T.J., 1987. Influence of interfacial waves in stratified gas-liquid flows. *AIChE J.* 33 (3), 444–454.
- Cheremisinoff, N.P., Davis, E.J., 1979. Stratified turbulent-turbulent gas-liquid flow. *AIChE J.* 25 (1), 48–56.
- Eck, B., 1973. *Technische Strömungslehre*. Springer, New York.
- Fisher, S.A., Pearce, D.L., 1979. A theoretical model for describing horizontal annular flows. *Two-phase Momentum, Heat and Mass transfer in Chemical, Process and Energy Engineering Systems*. Durst, Tsiklauri and Afgan. 1. 327–337.
- Grolman, E., Fortuin, J.M.H., 1995. The wavy-to-slug flow transition in inclined gas-liquid pipe flow. Presented at Two-Phase Flow Modelling and Experimentation, Rome, Italy. 1363–1370.
- Hart, J., Hamersma, P.J., Fortuin, J.M.H., 1989. Correlations predicting frictional pressure drop and liquid holdup during horizontal gas-liquid pipe flow with a small liquid holdup. *Int. J. Multiphase Flow* 15 (6), 947–964.
- Hoogendoorn, C.J., 1959. Gas-liquid flow in horizontal pipes. *Chem. Engng. Sci.* 9, 205–217.
- Kowalski, J.E., 1987. Wall and interfacial shear stress in stratified flow in a horizontal pipe. *AIChE J.* 33 (2), 274–281.
- Paras, S.V., Vlachos, N.A., Karabelas, A.J., 1994. Liquid layer characteristics in stratified-atomization flow. *Int. J. Multiphase Flow* 20 (5), 939–956.
- Spedding, P.L., Hand, N.P., 1995. Prediction of holdup and frictional pressure loss in two phase co-current flow. Presented at Two-Phase Flow Modelling and Experimentation, Rome, Italy. 573–581.
- Taitel, Y., Dukler, A.E., 1976. A model for predicting flow regime transitions in horizontal and near horizontal gas-liquid flow. *AIChE J.* 22, 47–55.
- Vlachos, N.A., 1997. Studies of stratified/atomization two-phase flow in a horizontal pipe. Ph.D. Thesis, Aristotle University of Thessaloniki, Chem. Eng. Dept., Greece.
- Vlachos, N.A., Paras, S.V., Karabelas, A.J., 1997. Liquid-to-wall shear stress distribution in stratified/atomization flow. *Int. J. Multiphase Flow* 23 (5), 845–863.